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ELECTRO MOTIVE

**NEW 144-AUTO FERRIES PROJECT**
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**3-DOC-008 Rev. -**
(Formerly Calculation P-0291-08)
(Centa 3-05905)

Contract 00-6679

# ENGINEERING CALCULATION

# HIGH SPEED SHAFTING
# DEFLECTION, BENDING STRESS & RESONANCE

Submitted by: The Centa Corporation

Date: March 30, 2006

Reviewed/Approved by:

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3/30/06
Date

## CALCULATION

TITLE: HIGH SPEED SHAFT DEFLECTION, BENDING STRESS AND RESONANCE

PURPOSE:

Determine the maximum shaft deflection for the maximum tube length and verify that the resulting bending stresses are within limits and that sufficient margin to shaft resonance exists.

SPECIFICATIONS AND ASSUMPTIONS:

- It is assumed that a calculation at maximum length will result in lower frequency, therefore shorter lengths will result in higher frequencies and therefore further from the operating speed range, resulting in a higher margin of safety.

The critical bending speed $n_{krit}$ can be calculated with the formula

$$\omega^2 = \left(\frac{\pi}{l}\right)^4 \cdot \frac{\hat{E}_x \cdot I_y}{\rho \cdot A}$$

further the $I_y$ second moment of area of a cylindrical circle and thin-walled tube calculates to:

$$I_y \approx \frac{\pi \cdot d_m^3 \cdot t}{8}$$

$$d_m = \frac{D_a + D_i}{2}$$

$$t = \frac{D_a - D_i}{2}$$

$$\omega_{krit} = \frac{\pi^2}{\sqrt{8}} \cdot \frac{d_m}{l^2} \cdot \sqrt{\frac{\hat{E}_x}{\rho}}$$

$$\omega = 2 \cdot \pi \cdot f$$

$$n = 60 \cdot f$$

$$n_{krit} = \frac{30\pi}{\sqrt{8}} \cdot \frac{d_m}{l^2} \cdot \sqrt{\frac{\hat{E}_x}{\rho}}$$

REFERENCES:

1. Given Data:

| | |
|---|---|
| For Coupling Type | CL-75 |
| Outer Diameter | $d_a = 240$ mm |
| Inner Diameter | $d_i = 226$ mm |
| Mean Diameter | $d_m = 233$ mm |
| Wall Thickness | $t = 7$ mm |
| Length of Tube | $l = 4898$ mm |
| Mass of Tube | $m_r = 197.01$ kg |
| Nominal Torque of Drive Line | $T_N = 24100$ Nm |
| Material | Steel |
| Proof Stress | $R_e = 235$ N/mm² |
| E-modul | $\hat{E}_x = 205000$ N/mm² |
| Specific Material | $\rho = 7850$ kg/m³ |
| Frequency | $f = 1/s$ |
| Meter | $m = 1000$ mm |
| Critical Angular Speed | $\omega_{krit}$ |

CALCULATION:

$$n_{krit} = \frac{30\pi}{\sqrt{8}} \cdot \frac{0{,}233m}{(4{,}898m)^2} \cdot \sqrt{\frac{205000 \cdot 10^6 \frac{N}{m^2}}{7850 \frac{kg}{m^3}}}$$

| | |
|---|---|
| Maximum Deflection | $f_n = 0.405$ mm |
| On position | $x = 2449.0$ mm |
| Natural Frequency of the Tube | $f_e = 27.6$ Hz |
| Resonance Speed | $n_{krit} = 1654$ rpm |

$$safety = \frac{n_{krit}}{n_{op}}$$

$$safety = \frac{1654}{930} = 1{,}78$$

| | |
|---|---|
| Calculated Torque & Bending Stress | sigma V = 72.1 N/mm² |
| Allowable Bending Stress | sigma A = 211.5 N/mm² |
| Resulting Load Factor | 34.1 % |

CONCLUSION:

The safety margins are satisfactory regarding bending and loads relating to the shaft resonance.